XIX. "On Curves of the Third Order." By the Rev. GEORGE SALMON, of Trinity College, Dublin. Communicated by ARTHUR CAYLEY, Esq. Received May 20, 1858.

(Abstract.)

The author remarks that his paper was intended as supplementary to Mr. Cayley's Memoir "On Curves of the Third Order" (Philosophical Transactions, 1857, p. 415). He establishes in the place of Mr. Cayley's equation, p. 442, a fundamental identical equation, which is as follows, viz. if substituting in the cubic U, $x+\lambda x'$, $y+\lambda y'$, $z+\lambda z'$ for x, y, z, the result is

$$U+3\lambda S+3\lambda^2 P+\lambda^3 U';$$

so that S and P are the polar conic and polar line of (x', y', z'), with respect to the cubic, viz.

$$3S=x'\frac{dU}{dx}+y'\frac{dU}{dy}+z'\frac{dU}{dz}; \quad 3P=x\frac{dU'}{dx'}+y\frac{dU'}{dy'}+z\frac{dU'}{dz'};$$

and if making the same substitution in the Hessian H, the result is

$$H+3\lambda\Sigma+3\lambda^2\Pi+\lambda^3 H',$$

so that Σ and Π are the polar conic and polar line of the Hessian—then the identical equation in question is

$$3(S\Pi-\Sigma P)=H'U-HU'.$$

And it follows that when (x', y', z') is a point on the cubic, the equation $U=0$ of the cubic may be written in the form

$$S\Pi-\Sigma P=0,$$

an equation which is the basis of the subsequent investigations of the paper. The author refers to a communication to him by Mr. Cayley, of an investigation of the equation of the conic passing through five consecutive points of the cubic, in the case where the equation of the cubic is presented in the canonical form $x^3+y^3+z^3+6lxyz=0$, and he shows that by the help of the above mentioned identity, the investigation can be effected with equal facility when the equation of the cubic is presented in the general form; and he establishes various geometrical theorems in relation to the conic in question. Finally, the author considers an entirely new question in the theory of cubics, viz. the determination of the points of a cubic, through which it is possible to draw an infinity of cubics having a nine-point

contact, or complete osculation, with the given cubic. It is shown that the points in question are those which are their own third tangentials, and this suggests the consideration of the new canonical form, $x^2y+y^2z+z^2x+2mxyz=0$, of the equation of the cubic; this inquiry, however, is not pursued in the paper.

XX. "Researches on the Foraminifera."—Part III. On the Genera *Peneroplis*, *Operculina*, and *Amphistegina*. By W. B. Carpenter, M.D., F.R.S. &c. Received June 17, 1858.

(Abstract.)

In his preceding memoirs, the author has shown that two very dissimilar types of structure present themselves among Foraminifera, one characterized by its simplicity, the other by its complexity. In the former, of which *Orbitolites*, *Orbiculina*, and *Alveolina* are typical examples, the calcareous skeleton does not present any definite indications of organization, but seems to have been formed by the simple calcification of a portion of the homogeneous sarcode-body of the animal; that sarcode-body is but very imperfectly divided into segments, the communications between the cavities occupied by these segments being very free and irregular; the form of the segments themselves, and the mode of their connexion, are alike inconstant; and even the plan of growth, on which the character of the organism as a whole depends, though preserving a general uniformity, is by no means invariably maintained. In the latter, to which *Cycloclypeus* and *Heterostegina* belong, the calcareous skeleton is found to present a very definite and elaborate organization. The several segments of the body are so completely separated from each other, that they remain connected only by delicate threads of sarcode. Each segment thus isolated has its own proper calcareous envelope, which seems to be moulded (as it were) upon it; and this envelope or shell is perforated with minute parallel tubuli closely resembling those of dentine, except in the absence of bifurcation; the partition-walls between adjacent segments are consequently double, and are strengthened by an intermediate calcareous deposit, which is traversed by a system of inosculating passages that seems properly to belong to it. The form of the segments, their mode of